Orbital Stability Analysis of Nigeria Sat-1 Spacecraft Using Predicted Two-Line Element Sets Data for 2004-2011

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Abstract

The planetary physics governing the launch of artificial satellite demands that its first orbit is a function of the first position in space (p_o), the velocity of its travel (v), the force field (f) and the mass (m). However, as time goes on, this initial orbital path of the satellite may be inevitably altered laterally and vertically, depending on the direction of the perturbing forces (gravitational and non-gravitational); which imposes some measures of drift from its original path. One of the strategic ways of monitoring the Orbit of an EOS is the use of predicted two-line element (TLE) sets for velocity, orbit inclination, eccentricity and coefficient of drag variation analyses. From the results of the study, NigeriaSat-1 showed a total orbital angular drift of about 00° 19' 03" of arc (equivalent to about 34.29km, about 4.3km/year), towards the earth's pole between 27th September, 2003 and 27th September 2010. The eccentricity of the Orbit ellipse of the NigeriaSat-1 was ***slightly*** *unstable and* ***irregular*** *in September 2004, which might have resulted in in-ordinate imaging of same ground area or swath on the earth within the period. An anomaly in the eccentricity was recorded on the 7th September (corresponding to GPS Epoch Day 250) 2004. The TLE set analysis shows that the NigeriaSat-1 Coefficient of Drag was small; hence not subjected to extreme vertical and lateral drag in orbit, which may not be unconnected with its light weight/mass of 98kg (micro-satellite). For sustainability of the Nigeria Space programme, periodic monitoring of the NigeriaSat-2 orbital status with the TLE set from outset of launch and completion of in-orbit callibration is imperative. This is because its mass of 270kg is almost three times that of NigeriaSat-1; hence would be more susceptible to drag and perturbing accelerations in its orbit.*

Key words: *Orbit, Stability, NigeriaSat-1, Two Line Element Sets Data*

1.0 Introduction

An orbiting satellite follows an oval-shaped path known as an ellipse with the body being orbited, called the primary, and located at one of two points called foci. An ellipse is defined to be a curve with the following property: for each point on an ellipse, the sum of its distances from two fixed points, called foci, is constant (Fig. 1.1). The longest and shortest lines that can be drawn through the centre of an

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ellipse are called the major axis and minor axis, respectively. The *semi-major axis* (a) is one-half of the major axis and represents a satellite's mean distance from its primary. *Eccentricity (e)* is the distance between the foci divided by the length of the major axis and is a number between **0** and **1**. An eccentricity of zero indicates a circle; which means that ***e*** is the measure of ellipse deviation from circularity. It is practically determined by the mean apogee and perigee distances along the elliptical orbit of the satellite.

A satellite orbiting a spherical earth moves with an acceleration of GM/r^2 towards the geocentre, subject to certain assumptions as below:

(i) The satellite must be small, both in size and weight. Most artificial micro-satellites easily satisfy this condition;

(ii) The earth must be spherical, its density must either be uniform or arranged in concentric uniform shells. The actual earth satisfies these conditions as a first approximation, but not all exactly;

(iii) There must be no disturbing forces, such as the attraction of the sun, moon and planets, air drag or radiation pressure. Actually, these forces do occur and, even though they are small, their combined effects are **not** negligible.

On these assumptions, a satellite will move in a fixed plane, in an unvarying space ellipse with the geocentre at one focus. Its rate of movement round the ellipse is not constant, but is such that the radius vector from the geocentre sweeps out equal areas in equal times (Kepler's Law of Planetary Motion). Therefore, the squares of the orbital period (T) of different satellites are proportional to the cubes of their semi-major axes (a). Therefore,

$$T = 2\pi a^{3/2}(GM)^{-1/2} \qquad (1.1)$$

$$T^2 = \frac{4\pi^2}{GM}a^3 \qquad (1.2)$$

Where a is given as

$$a = \frac{r_a + r_p}{2} = R_e + \left(\frac{h_a + h_p}{2}\right) \qquad (1.3)$$

Where r_a and r_p are the earth-centred radius of the satellite at apogee and perigee respectively, while Re = earth radius, h_a and h_p are altitudes of the satellites at apogee and perigee respectively. The magnitude of ***e*** as a counterpart of ***a*** in the definition of an orbit ellipse is given by equation (1.4a, b):

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{h_a - h_p}{2.R_e + h_a + h_p} \qquad (1.4a)$$

or

$$e = \frac{A}{GM_e} \qquad (1.4b)$$

Where A = Runge-Lenz vector, G = Gravitational Constant, given as $6.67259\text{x}10^{-11} \pm 0.00085\ m^3kg^{-1}s^{-2}$; M = mass of the earth, given as $5.974\text{x}10^{24}$kg [10].

1.1 Definition of Satellite Orbit

The satellite orbit can be defined by the classical set of Keplerian parameters, referred to the vernal equinox inertial coordinate axes. **In Fig. 1.1,** Ω is measured in the plane of the equator, and w and **v** in the plane of the orbit. The orbit and the position of the satellite at any instant can be described by six (6) orbital elements, otherwise known as Keplerian Orbital Elements of a Satellite. The orbital elements include: i, Ω, w, a, e and v. They are defined as:

- i = inclination of the plane of the orbit to that of the equator
- Ω= Right Ascension (RA) of the Ascending Node. Ascending node is the point of intersection of the satellite orbital sphere and the equator; the ascending node being when the satellite is passing from South to North. A progressive decrease of is known as a regression of the node.
- **p** = semi-latus rectum of the orbit, which is determined by $p = a(1-e^2)$, where a = semi-major axis of the orbit, **e** = eccentricity of the orbit; *determined by A/GM*, where G = Gravitational Constant, given as $6.67259 \times 10^{-11} \pm 0.00085\ m^3kg^{-1}s^{-2}$ [1]; *M*= mass of the earth, given as 5.974×10^{24}kg and A = Runge-Lenz vector [13; 10].
- **w** = angle between the ascending node and the radius vector to the satellite at perigee (when its distance is minimum). The point of perigee and apogee are known as apses; so, a progressive change in w is a rotation of the Apse; t_o = time when $v=0$ and the satellite is at perigee. It may sometimes be analytically convenient for t_o (dt) to be at the time of the ascending node.
- **v**= true anomaly
- Therefore, $w+v=u$ is the angle between the ascending node and the satellite at the celestial meridian; sometimes called the argument of latitude.

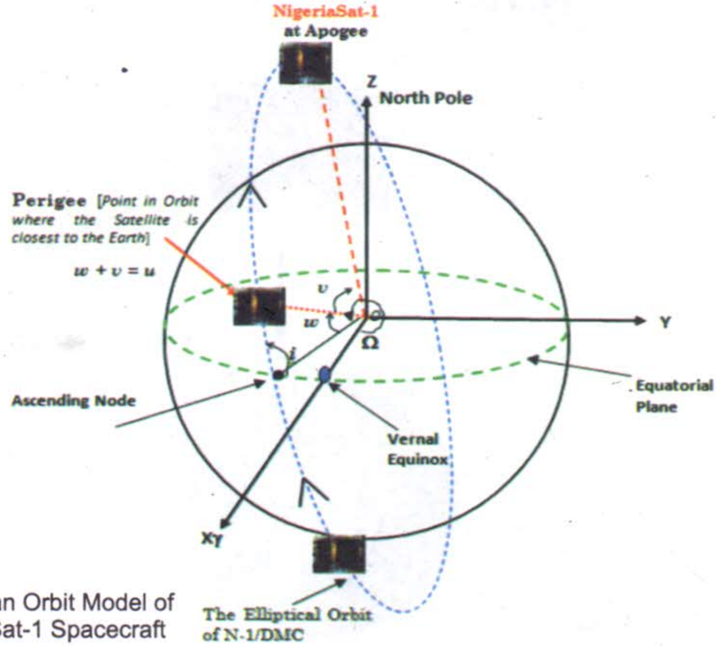

Fig. 1.1: Keplerian Orbit Model of NigeriaSat-1 Spacecraft

The eccentricity anomaly (E_a) is given by Kepler's equation as:

$$E_a - e.\sin E_a = M_a \text{ (radians)} \qquad (1.5)$$

Where e and M_a are eccentricity and mean anomaly respectively. The *true anomaly* *(v)* which is related to the *eccentric anomaly* *(E_a)* and the simplied form of equation (1.5) are given by [10] as equations (1.6) and (1.7) or (1.8).

$$v = \tan^{-1}\left(\frac{\sqrt{1-e^2}\sin E_a}{\cos E_a - e}\right) \qquad (1.6)$$

$$\cos E_a = \frac{\cos v + e}{1 + e.\cos v} \qquad \sin E_a = \frac{\sqrt{1-e^2}\sin v}{1 + e.\cos v} \qquad (1.7)$$

<u>or</u>

$$E_a = \tan^{-1}\left(\frac{\sqrt{1-e^2}\sin v}{\cos v + e}\right) \qquad (1.8)$$

Given the eccentricity (e) and the mean anomaly ($M_a \approx$ v) from the Two-Line Element (TLE) sets data or tracked orbit of the NigeriaSat-1 from two or three position vectors, the eccentric anomaly could be computed using equation (1.8), while equation (1.6) applies for the estimation of true/mean anomaly (v) if the eccentric anomaly (E_a) is known.

1.2 Satellite Orbit Degrading Forces

The graphic model of the disturbing forces operating on NigeriaSat-1 (N-1) spacecraft is illustrated by Fig. 1.2 below:

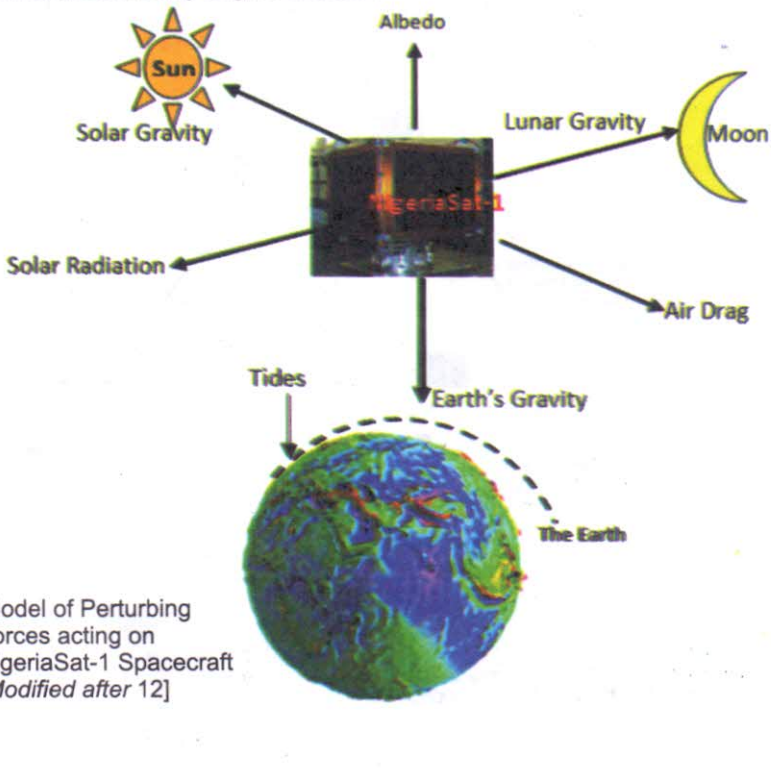

Fig. 1.2: Model of Perturbing Forces acting on NigeriaSat-1 Spacecraft [*Modified after* 12]

The gravitational and non-gravitational satellite orbit disturbing forces such as gravity ($\ddot{r}_g$), solar and terrestrial radiation pressure ($\ddot{r}_{sp}$) atmospheric drag ($\ddot{r}_d$), thermal heating and temperature ($\ddot{r}_T$), the direct effect of Sun and Moon ($\ddot{r}_{ls}$), earth and ocean tides, etc., are important parameters relevant in the orbit generation and monitoring of the satellites. The empirical evaluation of each component of this model is relevant in the orbit generation and improvement strategy for the satellites. The actual orbit of a satellite departs from the Keplerian orbit due to the various disturbing forces illustrated in Fig. 1.2, some of which are gravitational and non-gravitational in origin. These perturbing accelerations are actually responsibly for the modelled variations in the orbital elements with time.

The perturbed orbit can be viewed as the envelope of Keplerian Ellipse (Osculating Ellipse) which are defined at an instant by the current or instantaneous orbital elements a, e, i , w and M. The total perturbing acceleration on a near-earth satellite is given as:

$$\ddot{r}_t = \ddot{r}_g + \ddot{r}_{ls} + \ddot{r}_d + \ddot{r}_{sp} + \ddot{r}_T + \ddot{r}_e + \ddot{r}_o \quad (1.9)$$

Equation (7) is largely responsible for orbital drift and changes in the Keplerian parameters of spacecraft in orbit.

The planetary physics governing the launch of artificial satellite demands that its first orbit is a function of the first position in space (p_s), the velocity of its travel (v), the force field (f) and the mass (m) *[9; 2;* 10; 12]. However, as time goes on, this initial orbital path of the satellite may be inevitably altered laterally and vertically, depending on the direction of the perturbing forces (gravitational and non-gravitational). which imposes some measures of drift from its original path. Based on the orbit integration and orbit fitting methods, the influence of the characters of the gravity model, with different precisions, on the movement of low Earth orbit satellites, could be studied and monitored [3].

1.3 The Disaster Monitoring Constellation

The Disaster Monitoring Constellation (DMC) is a novel partnership initiative among Algeria, China, Nigeria, Turkey and the U.K. to achieve global coverage and daily satellite revisit. These satellites are in ***Sun synchronous orbits*** (SSO); hence their orbital plane precesses with the same period as the planet's solar orbit period. In such an orbit, a satellite crosses periapsis at about the same local time every orbit. In order to maintain an exact synchronous timing, it may be necessary to conduct occasional propulsive manoeuvres to adjust the orbit, which is better envisaged and managed if there are historical orbit monitoring exercises. In view of the foregoing, it is therefore important to monitor NigeriaSat-1, NigeriaSat-2 and NigeriaSat-X orbits and health in space as much as possible in order to model the satellite's drift trend for the purposes of thrusting and propulsive manoeuvres, among others where necessary.