Modelling Orthometric Heights from GPS-Levelling Observations and Global Gravity Model (EGM08) for Rivers State, Nigeria

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Abstract

One of the components of a geodetic framework is the height, which could be orthometric or ellipsoidal. Orthometric height, a height component referred to as the geoid, is required for engineering and geophysical projects, and other geospatial applications. The classical geodetic levelling method, used to determine orthometric height is tedious, time consuming and expensive. The method is almost impracticable in some parts of Nigeria, where the terrain is swampy and hostile; such as the Niger Delta region. However, this quantity can be determined from Global Positioning System (GPS) observations, if the geoidal undulation (N) is known from a national gravimetric geoid model. This method of obtaining orthometric height has been adopted by many countries of the world. This is because the approach is fast, less tedious in difficult terrain and relatively cheap when compared with the geodetic levelling technique. Unfortunately, this method cannot be used in Nigeria, because the gravimetric geoid in Nigeria has not been determined to substantial precision. This paper presents an alternative solution for the enrichment of the Nigerian height system using satellite technology and a global gravity model (GGM). In order to minimize the deviations or datum inconsistencies between the available height data, long-wavelength geoid errors at fifty-four (54) GPS/levelling collocated control points, a parametric corrective surface model was incorporated using a four, five, seven and eight-parameter datum shift transformation model. The results obtained at ten (10) test stations indicate the success of the approach in obtaining very accurate orthometric height and was thus recommended for solving the Nigerian height datum problem.

Keywords: Ellipsoidal height, orthometric height, EGM08, datum shift transformation and corrective surface

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1.0 Introduction

The need to avoid conflicts in land-use planning and the often disastrous consequences of uncoordinated environmental development requires maps which show not only the horizontal extent and content of a land area but also the levels of the terrain above a commonly understood surface. Maps, which show elevations of terrain points either as, contours or spot heights, are useful only when a common height system is adopted for the production of such maps. The common reference surface is usually referred to as vertical or height datum (HD). For countries with coastlines, the mean sea level (MSL) is often adopted as the reference for heights and it is established through measurements and analysis of several years of tidal data.

The need to have a unified vertical reference frame becomes so apparent in the era of information technology. Unification of vertical reference systems requires both geometrical and physical connections in the study area. Precise geometrical connection is provided from the regional GPS network while the physical relationship is to be provided from the regional or global precise geoid covering the whole of the study area.

The algorithm for orthometric height transfer using GPS has been widely presented and used. Its practical limitations are mostly due to datum bias inconsistencies and lack of precise geoid **[2 &3]**.

For over a century or so, traditional spirit levelling has been a technique of choice in the determination of orthometric heights. It is simple, the operation is effective, the method has remained basically unchanged, and yet it can achieve a remarkable precision. However, the observational time is too lengthy, making it a slow, labour-intensive, painstaking and costly operation. The method is almost impracticable in some parts of Nigeria, where the terrain is swampy and hostile; such as, the Niger Delta region. It is also a line operation whereby points whose heights are required need to be interconnected by a series of levelling lines. This makes levelling operation prone to many systematic errors which are difficult to detect and eliminate. Thus in recent times, many efforts have been made to develop alternative techniques and technologies to be incorporated into levelling to suit current needs.

Nowadays, the many benefits offered by space based measurements systems such as the Global Positioning System (GPS) have made it a suitable alternative over traditional levelling. Orthometric height determination using GPS is one of the possible applications that are gaining popularity. In recent times, the use of GPS for surveying, engineering and mapping applications is expanding at an astounding rate. This has prompted many state mapping agencies in Nigeria, including Rivers State to not only upgrade their existing reference systems to be GPS-compatible, but also to seriously look into the potential use of GPS in heighting. Perhaps, there is no other place in Nigeria where height datum problem is more pronounced than Rivers State, an oil-rich state which has become the headquarters of oil and gas activities. Because of the presence of different oil companies and private and public agencies, each operating independently, many different height systems exist which do not agree. Incidentally, the state government saw the need for the production of a new set of maps in order to be able to spread development projects more equitably. Akom Survey Services (AKOM), a mapping company was commissioned to carry out an orthophoto mapping and to establish fifty (50) 1st order GPS control points which could further be densified for local use. It was the execution of the control network that revealed the chaotic situation of the height system.

However, there are several problems associated with the use of GPS in vertical positioning. First, GPS gives elevations above a reference ellipsoid, World Geodetic System 1984 (WGS84), it gives heights which cannot be used directly with traditional orthometric height datum especially in determining the directions of water flow. Neither could the ellipsoidal heights be directly incorporated into the gravity based height systems.

This paper focuses on the construction of a corrective surface in the study area for further conversion of the ellipsoidal heights into orthometric heights within appreciable accuracies. Because appropriate gravimetric data are not available in Nigeria, satellite gravimetry based EGM08 solutions are fitted into GPS/levelling in the study area. Also presented, is the analysis of the performance of different corrector surfaces, with particular emphasis on datum inconsistencies, systematic effects and data accuracy.

2.0 Overview of Modelling Considerations for Corrector Surfaces

The fundamental relationship, to first approximation, that binds the heights from GPS i.e. ellipsoidal height and height with respect to a vertical (local) datum established from conventional spirit levelling and gravity data is given by numerous authors as [for example, 14,11,12 & 9]

$$h - H - N = q \tag{1}$$

where h is the ellipsoidal height, H is the orthometric height, N is the geoidal undulation and q is the offset of the vertical datum with respect to the geoid. In practice, the application of equation (1) is more complicated due to numerous factors, which cause discrepancies when combining the different height data sets **[2, 3, and10]**. The major part of these discrepancies is usually attributed to the systematic effects and datum inconsistencies, which can be described by a corrector surface model.

In practice, the various wavelength errors in the gravity solution may be approximated by different kinds of functions in order to fit the geoid to a set of GPS/levelling points through an integrated Least Squares (LS) adjustment. Several models can be used ranging from a simple linear regression to more complicated seven parameter similarity transformation models. The choice of the parametric form of the corrector surface model is not a trivial task. In fact, the list of potential candidates for the 'corrector' surface is extensive. Arguably, the selection process is arbitrary, unless some physical reasoning can be applied to the discrepancies between the GPS-derived geoidal heights, and the geoidal heights from the selected gravity field model (EGM08). The basic model for the corrector surface is as follows;

$$q_i = h_i - H_i - N_i = c_i^T x + v_i \tag{2}$$

and

$$\Delta N = N^{GPS} - N^{EGM08} = h_i - H_i - N_i = c_i^T x + v_i \tag{3}$$

Where, h_i,H_i, N_i are previously described, x is a $n\ x\ i$ vector of unknown parameters, and c is a $n\ x\ i$ vector of known coefficients, and vi denotes a residual random noise term. The standard deviation of adjusted values for the residuals traditionally is taken as the external indication of geoid model absolute accuracy [see **13**].

The parametric model $c_i^T x$ is supposed to describe the systematic errors and datum inconsistencies inherent in the different height data sets. Its type varies in form and complexity depending on a number of factors. In the past, researchers have often utilized a simple tilted plane-fit model, which in several cases has satisfied accuracy requirements **[1and10]**. However, as the achievable accuracy of GPS and geoid heights improves, the use of such a simple model may not be sufficient. The problem is further complicated because selecting the proper model type depends on the data distribution, density and quality, which varies for each case [see **10**]. This study focuses on the family of models based on the general 7-parameter similarity datum shift transformation [see **3, 4, 7, 8, and 16**]. Simplification leads to the classic 4-parameter model (model A), implemented as **[8 and 16]**:

$$c_i^T x = x_1 + x_2 \cos\phi_i \cos\lambda_i + x_3 \cos\phi_i \sin\lambda_i + x_4 \sin\phi_i \tag{4}$$

And

$$\Delta N = \Delta a + \Delta X_0 \cos\phi_i \cos\lambda_i + \Delta Y_0 \cos\phi_i \sin\lambda_i + \Delta Z_0 \sin\phi_i + v_i \tag{5}$$

Where ΔX_0, ΔY_0, and ΔZ_0, are the shift parameters between two parallel datum and Δa are the changes in semi-major axes of the corresponding ellipsoids, ϕ, λ are the latitude and longitude, respectively, of the GPS/levelling collocated points. By adding a fifth parameter, the extended classic 5-parameter model (model B) can be obtained. Thus;

$$c_i^T x = x_1 + x_2 \cos\phi_i \cos\lambda_i + x_3 \cos\phi_i \sin\lambda_i + x_4 \sin\phi_i + x_5 \sin^2\phi_i \tag{6}$$

And

$$\Delta N = \Delta a + \Delta X_0 \cos\phi_i \cos\lambda_i + \Delta Y_0 \cos\phi_i \sin\lambda_i + \Delta Z_0 \sin\phi_i + a\Delta f \sin^2\phi_i + v_i \tag{7}$$

Where ΔX_0, ΔY_0, and ΔZ_0, are the shift parameters between two parallel datum and Δa, Δf are the changes in semi-major axes and flattening of the corresponding ellipsoids. It should be noted that the parameters from such a 'datum shift transformation' do not represent the true datum shift parameters (translations, rotations and scale) because other long-wavelength errors inherent in the data (such as those in the geoid heights) will be interpreted as tilts and be absorbed by the parameters to some degree. A third form of a differential similarity transformation model (model C) is defined as **[3 and 16]**:

$$c_i^T x = x_1 \cos\phi_i \cos\lambda_i + x_2 \cos\phi_i \sin\lambda_i + x_3 \sin\phi_i + x_4 \left(\frac{\sin\phi_i \cos\phi_i \sin\lambda_i}{W_i}\right) +$$
$$x_5 \left(\frac{\sin\phi_i \cos\phi_i \cos\lambda_i}{W_i}\right) + x_6 \left(\frac{1 - f^2 \sin^2\phi_i}{W_i}\right) + x_7 \left(\frac{\sin^2\phi_i}{W_i}\right) \tag{8}$$